

HW SOL 7.1

March 5, 2018 1:50 PM

Name: _____

Date: _____

Section 7.1 What is Probability

1. Find the sample space for each of the following events:

i) Flipping two coins

HT TT
HH TH

ii) Rolling two dice

1,1 2,1 3,1 4,1 5,1 6,1
1,2 2,2 3,2 4,2 5,2 6,2
1,3 2,3 3,3 4,3 5,3 6,3
1,4 2,4 3,4 4,4 5,4 6,4
1,5 2,5 3,5 4,5 5,5 6,5
1,6 2,6 3,6 4,6 5,6 6,6

iii) Flipping 4 coins

HHHH HHTT TTHH TTTT
HHHT HHTH TTHH TTTT
HHTH HHTH TTHH TTTT
HTHH THHT TTHH TTTT
TTHH THHT TTHH TTTT

iv) Rolling a dice and then flipping a coin:

1T 1H
2T 2H
3T 3H
4T 4H
5T 5H
6T 6H

iv) A bag has 3 red marbles and 5 green marbles. Three marbles are chosen without replacement.

RRR GGG
RRG GGR
RGR GRG
GRR RGG

v) Drawing two cards from a deck without replacement.

52×51
 $= 2652$
outcomes

2. Find the probability of each event:

i) Rolling a dice and getting a 5

$\frac{1}{6}$

ii) Rolling two dice and getting a sum of 8

6,2 5,3 4,4 3,5 2,6 $\frac{5}{36}$

iii) Rolling two dice and getting a sum of 9 or more

9	10	11	12
4	3	2	1

$\frac{10}{36} = \frac{5}{18}$

iv) Rolling two dice and both numbers are odd

1,1 1,3 1,5
3,1 3,3 3,5
5,1 5,3 5,5
 $\frac{9}{36} = \frac{1}{4}$

v) Flipping 3 coins and getting at least 2 heads.

2H 1 3H
HHT HHT
HTH HTH
THH THH
 $= \frac{4}{8} = \frac{1}{2}$

vi) Flipping a coin 3 times and none of the outcomes are the same

~~HTH~~ HT □

3. Three letters are chosen in random one at a time without replacement. What is the probability that they will

3. Three letters are chosen in random one at a time without replacement. What is the probability that they will be in alphabetical order.

abc, cab
acb, cba
bac, bac

$$\frac{1}{6}$$

For every 3 letters, there are 6 ways to arrange them. only 1 out of 6 is in alphabetical order

4. A bag has 7 red marbles and a bunch of blue marbles. A marble is then chosen randomly. If the probability of getting a red marble is 28%, then how many blue marbles are there?

$$\frac{7}{100}$$

$$7(100) = 7(28) + 28(x)$$

$$\frac{7(72)}{72} = \frac{28x}{20}$$

$$x = \frac{8 \times 9}{4}$$

$$x = 18$$

5. Five cards are selected randomly from a deck of 52 cards. What is the probability that will it be a "Full House"?

A 2 3 4 5 6 7 8 9 10 J Q K

$$\frac{13 \times 12 \times 4C_3 \times 4C_2}{52C_5} =$$

6. Five cards are selected randomly from a deck of 52 cards. What is the probability that will it be a straight flush?

A 2 3 4 5 6 7 8 9 10 J Q K A

$$\frac{10 \times 4}{52C_5} = \frac{40}{52C_5}$$

7. Five cards are selected randomly from a deck of 52 cards. What is the probability that will it be two pairs?

① P P P P = same

④ $P_{(2,2)} = \frac{13C_2 \times 4C_2 \times 4C_2 \times 44}{52C_5}$

② # of ways to get 2 pairs

$$13C_2 \times 4C_2 \times 4C_2$$

③ # of cases for the same:

• 44 cases left.

8. Five cards are selected randomly from a deck of 52 cards. What is the probability that will it be only one pair?

① $\frac{13 \times 4C_2 \times 12C_3 \times 4^3}{52C_5}$

③ choosing 3 singles
 $12C_3 \times 4C_1 \times 4C_1 \times 4C_1$
 $= \frac{12 \times 11 \times 10}{1 \times 2 \times 3} \times 4^3$

④ $P_{(1,2,2)} = \frac{13 \times 4C_2 \times 12C_3 \times 4^3}{52C_5}$

② choosing pair:
 $13 \times 4C_2 = 13 \times \frac{4 \times 3}{1 \times 2} = 78$

9. Five cards are selected randomly from a deck of 52 cards. What is the probability that will it be a straight?

• STRAIGHT \neq STRAIGHT FLUSH.

$$P(x) = \frac{10 \times (52 - 4)}{52 C_5}$$

① A K Q J 10 9 8 7 6 5 4 3 2 A

10 ways to choose the numbers.

② Suits: $4 \times 4 \times 4 \times 4 \times 4 - 4$.

• Each card has 4 suits. • Minus the 4 cases where they can all be the same

10. In a classroom of 30 people, what is the probability that two or more people have the same birthday?

$$\frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365}$$

$$1 - \frac{365 P_{30}}{365^{30}} = P(x)$$

11. A coin is flipped 4 times getting either heads or tails. What is the probability of getting at least 3 tails?

AT LEAST 3 TAILS = 3T or 4T

$$\frac{4+1}{2^4} = \frac{5}{16}$$

12. Jim invited five friends to dinner: Amy, Brad, Cindy, Dave, and Eric, and only two people showed up. If each person is equally likely to show up, what is the probability of:

a. Amy was one of the people that showed up

$$\frac{4}{10} = \frac{2}{5}$$

AB BC CA DE
AC BD CE
AD BE
AE

b. Amy showed up but not Brad

$$\frac{3}{4}$$

c. Dave and Eric did not come

$$\frac{3}{10}$$

13. In the BC lotto 649, 6 numbers are chosen from 1 to 49 randomly, without replacement. If I bought a ticket, what is the probability that I will get all six numbers correct?

$$\frac{1}{49 C_6}$$

14. A parking lot has 16 spaces in a row. Twelve cars arrive, each of which requires one parking space, and their drivers choose their spaces at random from among the available spaces. Auntie Em then arrives in her SUV, which requires two parking spaces. What is the probability that she is able to park? Amc 2008

12 cars → 4 empty spaces.

1 CAR: $\binom{15}{1}$

2 cars: $\binom{14}{1} + \binom{13}{1}$

3 cars: $\binom{13}{2} + \binom{12}{2}$

4 cars: $\binom{12}{3} + \binom{11}{3}$

5 cars: $\binom{11}{4} + \binom{10}{4}$

6 cars: $\binom{10}{5} + \binom{9}{5}$

7 cars: $\binom{9}{6} + \binom{8}{6}$

8 cars: $\binom{8}{7} + \binom{7}{7}$

9 cars: $\binom{7}{8} + \binom{6}{8}$

10 cars: $\binom{6}{9} + \binom{5}{9}$

11 cars: $\binom{5}{10} + \binom{4}{10}$

12 cars: $\binom{4}{11} + \binom{3}{11}$

13 cars: $\binom{3}{12} + \binom{2}{12}$

14 cars: $\binom{2}{13} + \binom{1}{13}$

15 cars: $\binom{1}{14} + \binom{0}{14}$

16 cars: $\binom{0}{15} + \binom{-1}{15}$

16C4 = 1820

715

$$= \frac{1820 - 715}{1620}$$

3

15. Tina randomly selects two distinct numbers from the set $\{1, 2, 3, 4, 5\}$, and Sergio randomly selects a number from the set $\{1, 2, 3, \dots, 10\}$. What is the probability that Sergio's number is larger than the sum of the two numbers chosen by Tina? Amc

Tina	1	2	3	4	5
1		2	3	4	5
2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10

Sergio

$\frac{1}{10} \times \left[\frac{24}{25} + \frac{22}{25} + \frac{19}{25} + \frac{15}{25} + \frac{10}{25} + \frac{6}{25} + \frac{3}{25} + \frac{1}{25} + 0 \right] = \frac{90}{10 \times 25} = \frac{9}{25} //$

16. Jason rolls a fair regular octahedral die marked with the numbers 1 through 8. Then Andy rolls a fair six sided die. What is the probability that the product of the two rolls is a multiple of 3? AMC

	1	2	3	4	5	6	7	8
1			✓			✓		
2								
3	✓	✓	✓	✓	✓	✓	✓	✓
4			✓			✓		
5			✓			✓		
6	✓	✓	✓	✓	✓	✓	✓	✓

$\frac{24}{48} = \frac{1}{2} //$

17. A palindrome between 1000 and 10,000 is chosen at random. What is the probability that it is divisible by 7? AMC 12

① $\frac{1-9}{9-9} = 1 \times 1$
 $9 \times 10 = 90$
 palindromes
 B/W 1000-90,000

② $ABBA \rightarrow ABB - 2A$
 $100A + 10B + B - 2A$
 $98A + 11B = 49 \times 2 \times A - 11B$
 'A' can be any number 1-9.
 'B' must be '7' or '0'

ie: 1001 1771
 2002 2772
 3003 3773
 4004 4774
 5005 5775
 6006 6776
 7007 7777
 8008 8778
 9009 9779

12 unique

18. Challenge: Six points on a circle are given. Four of the chords joining pairs of the six points are selected at random. What is the probability that the four chords form a convex quadrilateral? Amc 1999

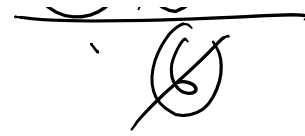
① $6C_2 = \frac{6 \times 5}{1 \times 2} = 15$ chords. ② how many ways to form a quadrilateral

$6C_4 = 6C_2 = \frac{6 \times 5}{1 \times 2} = 15$

③ $P(x) = \frac{15}{15 \times 7 \times 15} = \frac{1}{9!}$

$15C_4 = \frac{15 \times 14 \times 13 \times 12}{1 \times 2 \times 3 \times 4} = 15 \times 7 \times 13$
 ways to choose 4 chords

19. Challenge: Nine delegates went to a conference, 2 from Mexico, 3 from Canada, and 4 from United States. During the conference, three delegates fell asleep. Assuming that the three sleepers were determined randomly, what is the probability that exactly two of the sleepers are from the same country? AIME



AAA ABB ACC ADD AEE (15)
 AAB ABC ACD ADE
 AAC ABD ACE
 AAD ABE
 AAE

BBB BCC BDD BEE (10) (35)
 BBC BCD BDE
 BBD BCE
 BBE

(6)
(3)
(1)